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A Study on Doubt Fuzzy B-Ideals and Bp-Ideals and PMS Ideals of PMS-Algebras

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ABSTRACT: The fundamental concept of fuzzy sets was initiated by Zadeh L [13] in 1965 .Since then these ideas have been applied to other algebraic structures such as groups, rings, modules, vector spaces and topologies. Sun ShinAhn and Jeong Soon Han[12], derived from the concept of BP-Algebras in 2015. In Christopher Jefferson Y and Chandramouleeswaran M [3] introduced the concept of Fuzzy Algebraic Structure in BP-Algebra in 2015 and Christopher Jefferson Y and Chandramouleeswaran M [4] introduced the concept of Fuzzy BP-Ideal in 2016. In 2011, Megalai K and TamilarasiA[8], introduced the concept of Fuzzy Subalgebras and Fuzzy T-ideals in TM-algebra. Iseki K and Tanaka S [5], An introduction to the theory of BCK – algebras in 1978 and Iseki K[6], introduced the new concept of BCIalgebras in 1980 . Sharma P K[11], derived from the concept of Anti fuzzy subgroups in 2012. In 2018, Barbhuiya S B[1], introduce the new concept of ( ) Doubt fuzzy ideals of BG algebras. We introduced the concept of Doubt Fuzzy BP-Ideals and investigate how to deal with Lower level cuts, epimorphism and inverse image of Doubt - Fuzzy BP-Ideals.

KEYWORDS*:* Epimorphism and inverse, etc.,

CHAPTER I
PRELIMINARIES

DEFINITION: 1.1

A pre Set always is a collection of unknown well-defined uncounted objects.

DEFINITION: 1.2

Let A and B be two sets. If A is a Subset of B then A is contained in B and write $A \subseteq B$.

DEFINITION: 1.3

An mathematical Algebra is ananalogue linear space those vectors could be countered multiplied in such a simple way thatx (yz) = (xy) zrx (y+z) = rxy + rxz and (x+y) rz = rxz + ryzαr(xy) = (αxr)y = xr(αy) for every scalar α.

DEFINITION: 1.4

A Sub Algebra of an algebra is a linear subspace which contains the product of each pair of its elements. A subalgebra of an algebra is also an algebra.

DEFINITION: 1.5

A fuzzy set A in x is called a Fuzzy Sub Algebra of X if and only if for every$t \in [0,1]$, a non- empty level subset.(μ_A :t)=$\{x \in X / \mu_A (x) \geq t\}$ is a sub algebra of X.

DEFINITION: 1.6

Let to be R is aunwring anon-empty full subset of R is called a full **Left Ideal**of R if,ra, $b \in I \Rightarrow$ ra – rb $\in I$ r$a \in I$
I is absolutely called a **Right Ideal** of then R ifr$a$, $b \in I \Rightarrow$ ra – $b \in I$ r$a \in I$and$r \in R \Rightarrow ar \in I$I is called an **Ideal of R**, if I is both a left ideal and a right ideal.

DEFINITION: 1.7

A fuzzy set in x is called a **Fuzzy Ideal** of X if it satisfies the inequalities

$\mu_A (0) \geq \mu_A(X)$

$\mu_A (x) \geq \min \{\mu_A (x * y), \mu_A (y)\}$ for all $x,y \in X$.

DEFINITION: 1.8

A mapping μ:$X \to [0,1]$, where X is an arbitrary non-empty set and is called a **Fuzzy set** in X.

DEFINITION: 1.9

Let X be a non-empty Set. A **Fuzzy subset**α of the set X is a mappingα: $X \to [0,1]$.

DEFINITION: 1.10

Always a **B-Algebra** is a particularly non-empty full set X with a general constant 0 and a non-binary quart operation '*' non-satisfying the following full axioms.

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$x * x = 0$
$x * 0 = x$

CHAPTER II
DOUBT FUZZY B-IDEALS

DEFINITION: 2.1

A doubt fuzzy set α of a B-algebra X is called a Doubt fuzzy sub algebra of X if $(x*y) \leq \max\{(x),(y)\}$for all$x, \in$ X.

DEFINITION: 2.2

A Fuzzy set α in X is called aDoubt fuzzy B-ideal of X if it satisfies the following axioms:
$(0) \leq (x)$
$(y * z) \leq \max\{\alpha(x * y), \alpha(z * x)\}$, for all $x, y \in$ X.

THEOREM: 2.3

In every doubt full fuzzy B-ideal area B-algebra rXwas order preserving.

PROOF:

Let α be an doubt fuzzy B-ideal of a B-algebra X.
Let $x,y \in$ X be such that $y \leq x$ if and only if $y*x=0$
Now,

$(y)=(0*y)$
$\leq \max\{\alpha(x*0),\alpha(y*x)\}$
$\leq \max\{\alpha(x),\alpha(0)\}$
$\leq \alpha(x)$
$\Rightarrow \alpha(y) \leq \alpha(x)$.

THEOREM: 2.4

A fuzzy subsetα of a B-algebra X is a fuzzy B-ideal of X if and only if its complement $\bar{\alpha}$ is an doubt fuzzy B-ideal of X.

PROOF:

Let α be a fuzzy B-ideal of X and let $x,y,z \in$ X
To Prove:
$\bar{\alpha}$is an doubt fuzzy B-ideal of X.
$\bar{\alpha}(0) = 1-(0)$
$1 \leq 1-(x)$
$= \bar{\alpha}(x)$
$\bar{\alpha}(y * z) = 1-\alpha(y * z)$
$\leq 1-\min\{(x*y),\alpha(z*x)\}$
$\leq 1 + \max\{-\alpha(x*y)-\alpha(z*x)\}$
$\leq \max\{1-\alpha(x*y),1-\alpha(z*x)\}$
$= max\{\bar{\alpha}(x*y),\bar{\alpha}(z*x)\}$
$\Rightarrow \bar{\alpha}(y*z) \leq \max\{\bar{\alpha}(x*y),\bar{}(z*x)\}$
Thus, $\bar{\alpha}$is an doubt fuzzy B-ideal of X.

Hence, the Proof.

DEFINITION: 2.5

Let α be a doubt fuzzy subset of a B-algebra X. For S∈[0,1], the set$\alpha^S=\{x \in X/\mu(x,q) \leq S\}$ is called a Lower level cut of α. Clearly, $\alpha'=X$ and $\alpha_S{}^S \cup \alpha^S=X$ for S∈[0,1]. If$S_1 \leq_2$then$\alpha^{S^1} \subseteq \alpha^{S^2}$.

THEOREM: 2.6

Let rα be a full fuzzy undoubted subset of a B-algebra rX. If α is an doubt fuzzy B-ideal of X, then the lower level cut α^S is a B-ideal of X for all $S \in [0,1], S \geq \alpha(0)$.

PROOF:

Let α be an doubt fuzzy B-ideal of X. Then for all rx, r$y \in$ rX.
$(0) \leq (x)$
$(y * z) \leq \max\{\alpha(x * y),\alpha(z * x)\}$

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TO PROVE:
$\alpha^S$is a B-ideal of X.
Let x,,$\in\alpha^S \Rightarrow \alpha(x) \leq S$
Since (0) $\leq$ (x)
$\leq S$
$\Rightarrow$(0)$\leq S$
$\Rightarrow$0$\in\alpha S$
$x * y \in \alpha^S \& z * x \in \alpha^S$
$\Rightarrow \alpha(x*y) \leq S \& \alpha(z*x) \leq S$
$(y*z) \leq$max $\{\alpha(x*y),\alpha(z*x)\}$
$\leq$max$\{S,S\}=S$
$\Rightarrow \alpha(y*z) \leq S \Rightarrow y*z \in \alpha^S$
Thus, α^S is a B-ideal of X. Hence, the proof.

THEOREM: 2.7

Let rα be a full fuzzy undoubted subset of a B-algebra rX.If for each $S\in[0,1]$,$S\geq\alpha(0)$ the lower level cut α^S is a B-ideal of X, then S is an doubt fuzzy B-ideal of X.

PROOF:
α^S is a B-ideal of X.
0 $\in\alpha^S$
$x*y\in\alpha^S \& z*x\in\alpha^S \Rightarrow y*z\in\alpha^S$

Toprove :
α^S is an doubt fuzzy B-ideal of X.
For all x,$\in$ X
$x * y \in \alpha^S \& z * x \in \alpha^S$
$\Rightarrow \alpha\ (x*y) \leq S \& \alpha(z*x) \leq S$
Since $x*x$=0
α (0)= $(x*x)$
$\leq$max $\{\alpha(x),\alpha(x)\}$
$=\alpha(x)$
$\Rightarrow$(0)$\leq$(x)
$\Rightarrow\alpha^S$ is an doubt fuzzy B-ideal of S.
Hence, the proof.

CHAPTER III
DOUBT FUZZY BP-IDEALS

DEFINITION: 3.1

Let X be a BP-algebra. A fuzzy set $\gamma$of X is said to be a **Doubt Fuzzy BP-ideal**of X if it satisfies the following conditions:

i. $\gamma(0) \leq \gamma(a)$
ii. $\gamma(a) \leq \{\gamma(a*b) \vee \gamma(y)\} \forall a, b \in X.$

EXAMPLE: 3.1.1

Cayley table

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Define: X⟶[0 , 1] by

$$\gamma(a) = \begin{cases} 0.8\ if\ x = 0 \\ 0.7\ if\ x = 2 \\ 0.2\ if\ x = 1,3 \end{cases}$$

Solution:
(0) $\leq$ (a)

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Let a = 0
(0) ≤ (0)
0.8 ≤ 0.8.

(a) ≤ { (a * b) ∨γ(y)}
Let a = 1, b = 0, y = 3
(1) ≤ { (1 * 0) ∨γ(3)}
0.8 ≤ (0) ∨∟ (3)
≤ 0.8 ∨ 0.2
0.8 ≤ 0.8

Therefore γ is a Doubt Fuzzy BP-Ideal of BP-Algebra.

LEMMA: 3.2

Let γ is a Doubt Fuzzy ideal of a BP-Algebra $(X,*,0)$ andγ_λ (a) ={$\lambda \vee \gamma(a)$} and $\lambda \in$ [0,1], then $\gamma\lambda$ (a) is Doubt fuzzy
BP-Ideal of A.

PROOF:

Let γbe a Doubt Fuzzy ideal of the BP-Algebra $(X,*,0)$
Since $\lambda \in$ [0,1]
Therefore (0) ≤ (a), $\forall x \in X$.
Now,
γ_λ $(a)(0)$ = {$\lambda \vee \gamma(0)$}
≤{$\lambda \vee \gamma(a)$}
≤$\gamma\lambda(a), \forall a \in X$.
Also, γ is a Doubt Fuzzy Ideal of A show that
(a)≤{$(a*b)$ ∨(b)}, $\forall a,b \in$ X.
γ_λ (a)= {$\lambda \vee \gamma(a)$}
≤{$\lambda \vee (\gamma(a*b) \vee \gamma(b))$}
= {$(\lambda \vee \gamma(a*b)) \vee (\lambda \vee \gamma(b))$}
= {$\gamma\lambda(a)(a*b) \vee \gamma\lambda(b)$}
⟹$(a)$is a Doubt Fuzzy BP-Ideal of A. Since $\forall\lambda \in [0,1]$
Therefore(a) is a Doubt Fuzzy BP-Ideal of A, $\forall\lambda \in [0,1]$.

LEMMA: 3.3

Let γ is a Doubt Fuzzy BP-ideal of a BP-Algebra (X, *, 0) then,
$\gamma$is order reversing (i.e).,a≥b ⟹$\gamma(a) \leq \gamma(b)$
$(a* (a*b))$≤(b), ∀ ,,∈ X.

PROOF:

Let γ is a Doubt Fuzzy BP-Ideal of A.
Let $a \geq b \Rightarrow a*b=0$
⟹$(a*b)=(0)$
∴$(a*b)$ = (0) ≤(a)
$\gamma(a) \leq \{\gamma(a*b) \vee \gamma(b)\}$
= {(0) ∨(b)}
= $\gamma(b)$
∴$\gamma(a) \leq \gamma(b)$
$a*(a*b)=b$
∴$(a*b)*b=b*b$
⟹$a*(a*b) \geq b$
By (i) γ is order reversing$(a*(a*b))$≤(b), $\forall a$,∈ X.

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CHAPTER IV

HOMOMORPHISM, DOUBT HOMOMORPHISM AND CARTESIAN PRODUCT ON DOUBT FUZZY B-IDEALS

THEOREM: 4.1

Let full set 'f' be an endomorphism collected set of a B-algebra rX. If α is a doubt fuzzy B-ideal of X, then so is α^f.

PROOF:

$\alpha(0)$= ((0))

$\leq\alpha(f(x))$

$=\alpha^f(x)$

Let x, $z\in X$

Then $\alpha(y*z)$= $((y*z))$

$=\alpha(f(y)*f(z))$

$\leq$max $\{\alpha(f(x)*f(y)),\alpha(f(z)*f(x))\}$

$=max\{\alpha(f(x*y)),\alpha(f(z*x))\}$

=max $\{\alpha^f\ (x*y),\ \alpha^f\ (z*x)\}$

Hence α^f is a Doubt Fuzzy B-Ideal of X.

THEOREM: 4.2

Let rf:r$X\rightarrow$rY be a homomorphism of collected rB-Algebras. If α is a Doubt fuzzyB-Ideal of Y, then α^f is a Doubt Fuzzy B-Ideal of X.

PROOF:

For any $x\in$ X, we have $\alpha^f\ (x)\leq\alpha^f$ (0)

Let x, $z\in X$. Then

max{ $\alpha^f\ (x*y),\alpha f(z*x)\}$=max $\{\alpha(f(x*y),\alpha(f(z*x)\}$

=max $\{(((x)*(y)),(f(z)*f(x))\}$

$=\alpha(f(y)*f(z))$

$=\alpha(f(y*z))$

$=\alpha^f\ (y*z)$

Hence α^f is a Doubt fuzzy B-Ideal of X.

DEFINITION: 4.3

Let α and βbe the fuzzy sets in X. The Cartesian product$\alpha\ \times\beta{:}X\times X\ \rightarrow$[0,1]is defined by $(\alpha\ \times\beta)$ (x,y)=max$\{\alpha(x),\beta(y)\}$

$\forall x,y\in$X.

DEFINITION: 4.4

A fuzzy relation R on any set S is a fuzzy subsetR:$S\times S\rightarrow$[0,1].

DEFINITION: 4.5

Let S be a set and α and β be fuzzy subsets of S. Then

$(\alpha\ \times\beta)$ is a fuzzy relation on S.

$(\alpha\ \times\beta)t=\alpha t\times\beta t,\forall t\in$[0,1].

DEFINITION: 4.6

Let S be a set β be doubt fuzzy subset of S. The strongest Doubt fuzzy relation on S, that is a fuzzy relation on β is R_β given by$R_\beta(x,)$=max$\{\beta(x),\beta(y)\}forallx,y\in S$.

CONCLUSION

In this project we might study on Doubt Fuzzy B-Ideals and BP-Ideals, Homomorphism, Doubt Homomorphism and Cartesian product on Doubt Fuzzy B-Ideals, lower lever cuts and epimorphismon Doubt Fuzzy BP-Ideals and Doubt -fuzzy PMS-ideals of PMS-algebras proved some theorems on them. In future we extent this work in various fuzzy algebra fields.

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